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December 20, 2002

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A Self-Calibrating Multi-Band Region Growing Approach to Segmentation of Single and Multi-Band Images

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Abstract

Image segmentation transforms pixel-level information from raw images to a higher level of abstraction in which related pixels are grouped into disjoint spatial regions. Such regions typically correspond to natural or man-made objects or structures, natural variations in land cover, etc. For many image interpretation tasks (such as land use assessment, automatic target cueing, defining relationships between objects, etc.), segmentation can be an important early step.

Remotely sensed images (e.g., multi-spectral and hyperspectral images) often contain many spectral bands (i.e., multiple layers of 2D images). Multi-band images are important because they contain more information than single-band images. Objects or natural variations that are readily apparent in certain spectral bands may be invisible in 2D broadband images. In this paper, the classical region growing approach to image segmentation is generalized from single to multi-band images. While it is widely recognized that the quality of image segmentation is affected by which segmentation algorithm is used, this paper shows that algorithm parameter values can have an even more profound effect. A novel self-calibration framework is developed for automatically selecting parameter values that produce segmentations that most closely resemble a calibration edge map (derived separately using a simple edge detector). Although the framework is generic in the sense that it can imbed any core segmentation algorithm, this paper only demonstrates self-calibration with multi-band region growing. The framework is applied to a variety of AVIRIS image blocks at different spectral resolutions, in an effort to assess the impact of spectral resolution on segmentation quality. The image segmentations are assessed quantitatively, and it is shown that segmentation quality does not generally appear to be highly correlated with spectral resolution.

1. Introduction

Image segmentation is the process of extracting regions by dividing an image into disjoint sets of pixels that belong together. The input is a 2D or 3D image of pixel values, and the output is a *region map* (a 2D image in which each pixel is labeled with the integer-valued ID of the region to which it was assigned). Edges can be extracted as boundaries between regions in the region map. Image segmentation is a critical early step in a number of important applications and problem domains, including image understanding, automatic target cueing (automatically separating objects of interest from complex backgrounds), land use classification, etc.

Several families of image segmentation algorithms have been developed over the last several decades, including region growers, pixel classifiers ([Bouman1994], [MacQueen1967], [Ball1965], [Masson1993]), deformable model-based methods ([Kass1987], [McInerney1995], [Gunn1995], [Sethian1999]) and morphological methods ([Goutsias2000]). Region growers assign pixels to regions by searching local neighborhoods centered on pixels that already belong to the region for unassigned pixels that are spectrally similar to the *seed pixel* (i.e., the first pixel

that was assigned to the region). Global search algorithms define the search neighborhood to be the entire image, whereas local search algorithms are more efficient because they use small (typically 3x3) search neighborhoods. Region growers have historically been applied mostly to single-band (2D) images ([Muerle1968], [Zucker1976]). However, remotely sensed images (notably multi-spectral and hyperspectral images) often contain more than one spectral band (i.e., they are 3D images that contain multiple layers of 2D images). Multi-band imaging sensors were developed to detect spectral variations that occur in narrow bands that are obscured or invisible in broad-band images. They also have the capacity to sense spectral signatures of objects and physical phenomena, which is important because spectral shapes can provide greater discriminatory power than the mean spectral values (scalars) stored in pixels from single-band images. This paper provides a simple generalization of the classical region growing approach to image segmentation from single to multi-band images (see 3).

In this paper, a region growing approach to image segmentation was chosen because of its many favorable properties. When choosing an image segmentation algorithm, one should, in addition to segmentation quality, take several additional factors into account:

- requirements to make prior assumptions or to have prior knowledge
- applicability to both single and multi-band images
- ability to isolate objects with spatially separated parts from the background
- computational efficiency

Region growing algorithms measure favorably against these criteria. Region growers do not require the user to supply an estimate for the number of regions or spectral classes, and although they have historically been applied mostly to single-band images, they can be easily generalized to multi-band images (see Section 3). By increasing the size of the local search neighborhood, region growers have the ability to separate objects with spatially separated parts from the background. However, when a small search neighborhood is used (the usual case), region growing is among the most computationally efficient of image segmentation algorithms, with computational complexity that grows more or less linearly with the number of pixels and the number of spectral bands.

The fifth factor, segmentation quality, is more complex than the other four factors because it is affected not only by which segmentation algorithm is used, but also by

- pre-processing
- algorithm parameter values
- spectral resolution

In fact, the first two factors can have a more profound impact on segmentation quality than choice of segmentation algorithm (despite the fact that most of the community's effort has gone into development of competing segmentation algorithms). In Section 4, a novel self-calibration framework that automatically selects parameter values for segmentation algorithms is introduced. The framework is generic in that it can imbed any core segmentation algorithm, although in this paper, the self-calibration framework is only demonstrated with multi-band region growing. The self-calibration strategy is to select algorithm parameter values that produce a segmentation that most closely resembles an independently derived calibration edge map. In Section 5, the framework is applied to a variety of AVIRIS image blocks at different spectral resolutions, in an effort to assess the impact of spectral resolution on segmentation quality.

2. Block-Wise Processing and Block Pre-Processing

Before proceeding further, it is important to discuss block-wise processing and image block pre-processing. Large images (especially large images with many spectral bands) can quickly saturate the random access memory available to desktop computers and multi-processor suites. By processing images in blocks, image segmentation can scale to images of arbitrary size, spatially adapt to changes in the scene, and be carried out with parallel processors to increase the pixel throughput rate. However, regions that span borders between adjacent non-overlapping blocks will be split along block borders. This problem can be addressed by allowing successive blocks to overlap. Although this increases the number of blocks that must be processed, it handles regions less than one half of a block width in extent that might be split along borders between adjacent blocks.

One important goal of image segmentation is to emulate human cognitive abilities without involving a human analyst. Even human analysts enhance image blocks before attempting to interpret them. Likewise, it is important to automatically enhance each block before the computer attempts to segment it. This automatic enhancement should produce an image that a human analyst could look at and correctly interpret. In this paper, each image block is range-clipped and then quantized to 8 bits (since the AVIRIS images are $b > 8$ bit images). Each block is range-clipped to the p and $100\text{-}p$ percentiles of its band-averaged pixel values (the clipped range is extended to 256 when the difference between these percentiles is less than 256). For suitably small p (say $p = 1$, as in this paper), range-clipping enhances the brightness and contrast of the image block by removing statistical outliers. The process of discarding some of the raw data when image blocks are clipped and quantized is important in the quest to emulate human cognitive abilities.

3. Multi-Band Region Growing

The *Multi-Band Region Growing (MBRG)* algorithm for image segmentation assigns a pixel to a region only if it is spectrally similar to the seed pixel. MBRG thus extracts regions that possess a high degree of spectral homogeneity, i.e., the segmentations are not based on other factors, such as texture.[2] This emphasis is consistent with the goal of isolating man-made objects or targets of interest (since they tend to have largely homogeneous parts) from potentially complex backgrounds.

To be more precise, MBRG segments images by employing the local search operator $W_{i,j}(i\acute{},j\acute{})$ to grow regions from *seed pixels (i,j)*. MBRG is completely characterized by its local search operator. Pixel $(i\acute{},j\acute{})$ has been previously assigned to the region $R_{i,j}$ grown from seed pixel *(i,j)*, and it lies at the center of a local search neighborhood. Pixels *(m,n)* that have not been previously assigned to any region are added to $R_{i,j}$ if they lie within the local search neighborhood of pixel $(i\acute{},j\acute{})$ and are spectrally similar to seed pixel *(i,j)*. Mathematically, $W_{i,j}(i\acute{},j\acute{})$ can be viewed either as a unary operator whose only operand is the ordered pair $(i\acute{},j\acute{})$, or as a set of ordered pairs *(m,n)*. In general, pixel *(m,n)* is added to the region $R_{i,j}$ grown from seed pixel *(i,j)* if for any pixel $(i\acute{},j\acute{}) \in R_{i,j}$, $W_{i,j}(i\acute{},j\acute{})$ contains *(m,n)*, and *(m,n)* was not previously assigned to another region.

---

[2] However, they could be, simply by first transforming an image of spectral values to an image of values of interest, such as textural values.

MBRG models its local search operator as the intersection of two sets, one which imposes spectral constraints, and another which imposes spatial constraints on the pixels (m,n) that are assigned to $W_{i,j}(i',j')$. Mathematically,

$$(1) \qquad W_{i,j}(i',j') = W_{spectral}(i,j) \ \cap \ W_{spatial}(i',j')$$

$W_{spectral}(i,j)$ is the set of all pixels that are spectrally close to pixel (i,j). $W_{spatial}(i',j')$ is the set of all pixels that are spatially close to pixel (i',j'). A pixel (m,n) is thus assigned to $W_{i,j}(i',j')$ if it is both spectrally close to pixel (i,j) and spatially close to pixel (i',j').

The spectral constraint set (or spectral component) $W_{spectral}(i,j)$ is based on spectral distances between a seed pixel (i,j) and pixels $(m,n) \neq (i,j)$. Consider a single or multi-band image with K spectral bands that contains pixel spectra $x_{i,j} \stackrel{D}{=} [x_{i,j}(0),...,x_{i,j}(K\text{-}1)]^T$ with K spectral samples. $x_{i,j}(k)$ will refer to spectral sample k (the spectral band index) of the pixel with (row,column) coordinates (i,j). The squared spectral distance between pixels (i,j) and (m,n) can be defined as the squared norm of the difference between spectral vectors averaged over all bands, namely

$$(2) \qquad d_{i,j}^2(m,n) = || x_{i,j} - x_{m,n} ||^2 / K$$

This metric is inexpensive to compute, noise resistant (because it averages samples across all spectral bands), and reversible (i.e., $d_{i,j}(m,n) = d_{m,n}(i,j)$). For MBRG,

$$(3) \qquad W_{spectral}(i,j) = \{(m,n): d_{i,j}(m,n) < d* \}$$

where $d*$ is the spectral distance threshold.

The spectral similarity measure in equation (2) operates over all K spectral bands, so it may not be particularly sensitive to significant spectral variations that are limited to specific subbands. To further exploit the improved spectral resolution associated with images that have large numbers of spectral bands (such as hyperspectral images), $d*$ can be allowed to vary as a function of spectral band. Spectrally adaptive MBRG results when the decision rule in equation (3) is applied separately to multiple subbands. The image is first partitioned into B adjacent subbands $b = 1,...,B$. Then for spectrally adaptive MBRG,

$$(4) \qquad W_{spectral}(i,j) = \{(m,n): d_{i,j,b}(m,n) < d* \ \forall b\}$$

Equation (4) is similar to equation (3), except $d_{i,j,b}(m,n)$ is the spectral distance between pixels (i,j) and (m,n) within subband b.

The spatial constraint set (or spatial component) of the local search operator for MBRG is

$$(5) \qquad W_{spatial}(i',j') = \{(m,n) \neq (i',j'): |m\text{-}i'|, |n\text{-}j'| \leq w\}$$

This is the set of pixels in the square local search neighborhood of width $2w+1$ centered on pixel (i',j'), exclusive of (i',j'). If $w = 1$, these are the pixels that are 8-connected to pixel (i',j'). $w = 1,2,...$ is a *connectivity relaxation parameter*. The degree of spatial connectivity is relaxed by

increasing *w*. Region growing can be extended to produce regions with spatially disconnected parts by using a value of *w* greater than *1*. *w = 1* thus corresponds to local search over 3x3 neighborhoods, and it produces a segmentation that contains only simply connected regions. *w* ® ¥ corresponds to global search, which is tantamount to supervised pixel classification based on a Euclidean spectral distance metric, with spectral class centers chosen in row-order from among unassigned pixels. Mathematically, $W_{i,j}(i¢j¢) \rightarrow W_{spectral}(i,j)$ as *w* ® ¥.

One unfortunate artifact of most image segmentation algorithms (including MBRG) is that they tend to leave behind a significant number of small regions, that typically correspond either to "transition" pixels (such as edges along borders between regions), or to objects that are too small to be resolved at the spatial resolution of the image. Certain image segmentation algorithms based on supervised pixel classification address this problem by iterating from fine to coarse spatial resolution (i.e., they are multi-scale) [Bouman1994]. However, MBRG addresses this problem by merging all regions of less than a prescribed size *n > 0* (spatially connected or not) into one region. This tends to significantly reduce the number of regions. The merged region is then segmented into spatially connected regions. Finally, all remaining regions of less than a prescribed size (spatially connected or not) are merged into the smallest connected region. The net result of this is that all regions with less than *n* pixels are eliminated.

The regions extracted with the MBRG algorithm are disjoint, and they collectively cover the entire image (i.e., they form a complete set). Although MBRG segmentation is strictly seed pixel dependent, the regions tend to remain intuitively reasonable, independent of how the seed pixels are chosen.

4. Generic Framework for Automatic Self-Calibration

A novel framework for automatically selecting values for segmentation algorithm parameters is introduced in this Section. The framework is spatially adaptive since parameter values can vary for different image blocks. The framework is generic in the sense that it can imbed *any* core segmentation algorithm (although in this paper, the framework is only demonstrated with the MBRG algorithm). The framework is based on *self-calibration*, i.e., segmentation is performed at each of several parameter values, and the region map that is most consistent with an independently and automatically generated *calibration edge map* is used.

A calibration edge map can be generated separate from segmentation by applying an independent edge detector to image blocks that have been pre-processed as in Section 2. Edge detectors often treat edge pixels as local maxima in intensity gradient images. Intensity gradients can be estimated using gradient operators, such as Sobel operators. Edge strength can be estimated by computing the largest magnitude of difference in band-averaged pixel intensity between the edge pixel and each of its 8 neighbors. An edge pixel is eliminated if its strength lies beneath a perceptual difference threshold (say 16). If the number of edge pixels exceeds a prescribed fraction of the total number of pixels (say 1/3), the weakest edge pixels in excess of that number are eliminated. Finally edge segments with less than a prescribed number of pixels (say 5) can be eliminated. This edge detection procedure is particularly robust. It returns few (if any) edges when the image is bland, it limits edge clutter when the image is busy, and it produces intuitive results on images that are somewhere between bland and busy, even if the edges are faint. The exact values of the edge detection parameters are not important, as long as the difference threshold is consistent with the human visual perception and the fractional limit on the number of edge pixels is not too small. The objective is not to generate one specific edge map, but to automatically and consistently produce an edge map that is consistent with human visual perception (at present, it is much easier to produce such an edge map than it is to produce a region map that is consistent with human visual perception).

The self-calibration framework requires a measure of disparity between region maps and calibration edge maps. Consider a region map R and an associated binary border map B in which pixels of value 1 correspond to borders between different regions (specifically, $B(i,j) = 1$ if $R(i,j) \neq R(i-1,j)$ or $R(i,j-1)$). Let E be the calibration edge map with edge pixels of value 1 on a background of zeros (R, B and E all have the same number of rows and columns). The disparity D_{BE} between R and E is given by

$$
(6) \qquad D_{BE} = \begin{cases} 0 & n_B = n_E = 0 \\ 1 & n_B \text{ or } n_E = 0 \text{ but not both } 0 \\ (n_{BE} + n_{EB}) / (n_B + n_E) & n_B, n_E \neq 0 \end{cases}
$$

In equation (6), N_B is the number of boundary pixels in B, N_E is the number of edge pixels in E, N_{BE} is the number of boundary pixels in B that are not associated with an edge pixel in E, and N_{EB} is the number of edge pixels in E not associated with a boundary pixel in B. In other words, $0 \leq N_{BE} \leq N_B$ is the number of unassociated boundary pixels in B, and $0 \leq N_{EB} \leq N_E$ is the number of unassociated edge pixels in E, from which it can be seen that $0 \leq D_{BE} = D_{EB} \leq 1$. A boundary pixel in B is said to *correspond* to a closest edge pixel from E and vice-versa. Two pixels are said to be *associated* if they correspond and the distance between them (in pixels) is no greater than some *association distance* $d_a \geq 0$ ($d_a = 1.5$ was used in this study, in which case two pixels must be identical or 8-connected in order to be associated). N_{BE} can be quickly computed as the number of boundary pixels in B for which the *distance transform (DT)* of E (namely DT_E) $> d_a$ ($DT_E(i,j)$ represents the (Euclidean) distance in pixels between pixel (i,j) and the nearest nonzero pixel in E). Similarly, N_{EB} is the number of edge pixels in E for which $DT_B > d_a$. An efficient algorithm for computing error-free DT's of arbitrary bitmaps based on a variety of distance measures (including Euclidean distance) is given in [Paglieroni1992]. This algorithm has a provision to produce DT's that are clipped to a proximity threshold of d_a.

Fig.1 provides a graphical depiction of the generic self-calibration framework. Under-segmented images (coarse region maps with overly large and too few regions) and over-segmented images (fine region maps with excessively small and too many regions) corresponding to sub-optimal segmentation algorithm parameter settings are shown, along with their associated boundary maps and DT's. The "optimal" segmentation (corresponding to the algorithm parameter setting for which the disparity between the segmentation boundaries and the calibration edge map is minimal) is also shown. For MBRG, the algorithm parameter setting is the spectral distance threshold $d*$ in equation (3) (measured in units of gray-scale intensity quantized to 8 bits). In practice, only a few parameter settings are needed (for this paper, seven $d*$ values of 8 through 44 in increments of 6 were used). In some cases, the minimal disparity lies within an extended valley of parameter settings, i.e., there are a number of parameter settings that produce comparably low disparities. Therefore, all parameter settings that produce disparities that are within 10% of the minimal disparity are tabulated, and the median of these parameter settings is chosen as the "optimal" setting.
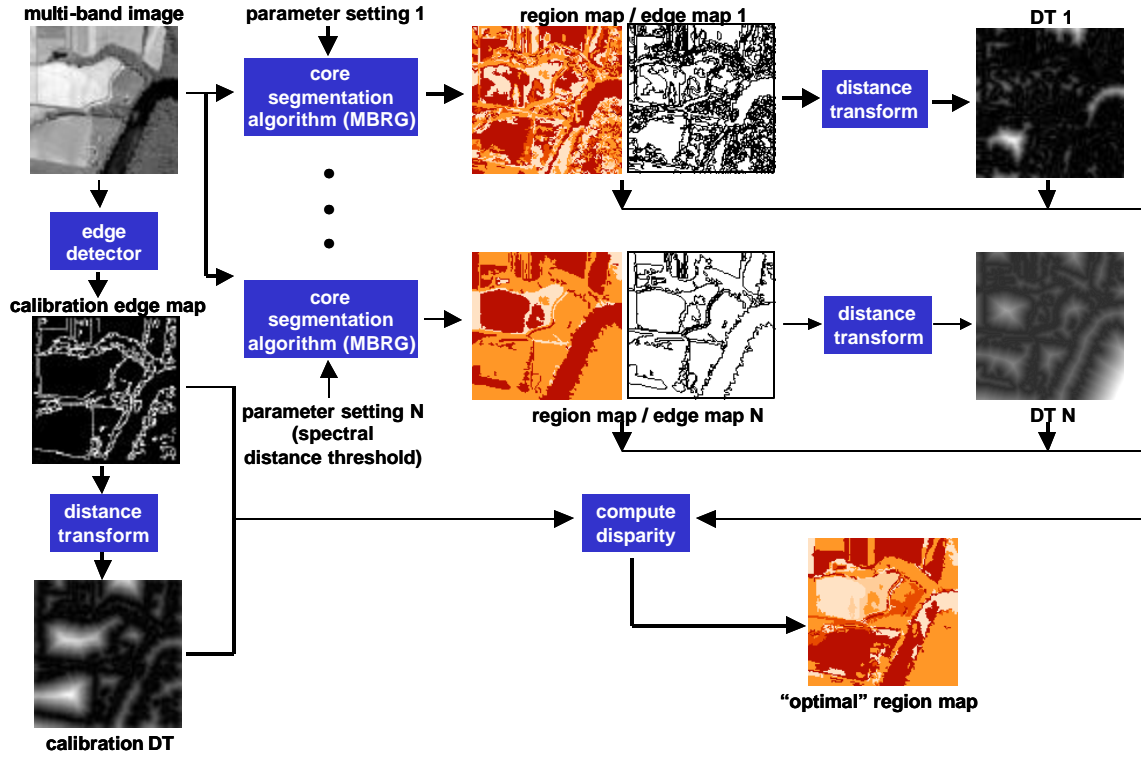
Fig.1   Graphical depiction of the generic self-calibration framework for image segmentation.

5. Examples and Experimental Results

In what follows, all images were 128 x 128 blocks taken from larger AVIRIS images (AVIRIS images have 224 spectral bands). All image blocks were pre-processed, as described in Section 2. All region maps were "cleaned" by removing all regions with less than 5 pixels. All region growing was based on a 3x3 local search neighborhood. In the displayed region maps, pixels that are the same color *and* which lie in the same 8-connected region belong to the same region (i.e., 2 blobs that are not 8-connected correspond to 2 different regions, even if they have the same color).

Fig.2 demonstrates the effect that changing the MBRG parameter setting has on region maps using a rural AVIRIS image block blurred down to a spectral resolution of 16 bands. Fig.3 shows how disparity between region map and calibration edge map varies with MBRG parameter setting for the image block in Fig.2 at three different spectral resolutions.
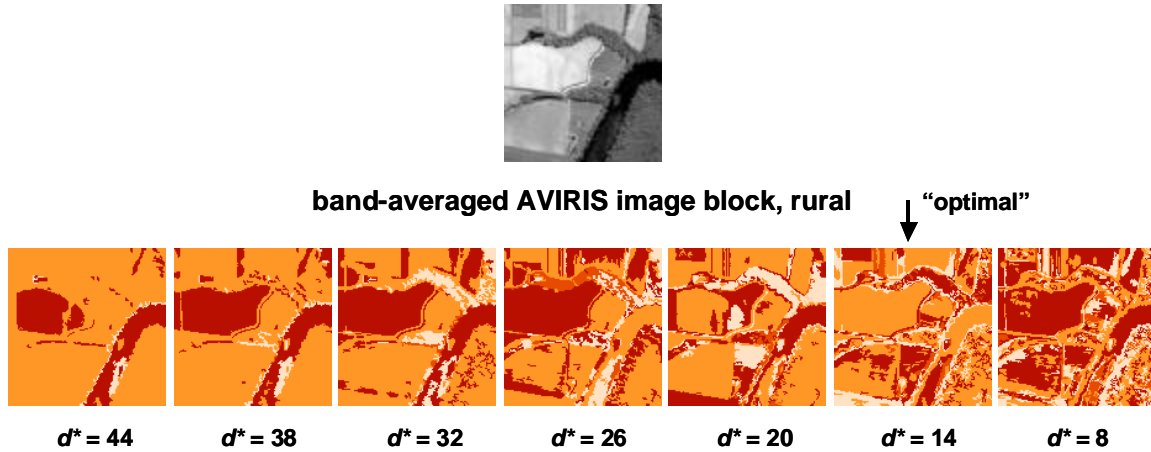
band-averaged AVIRIS image block, rural   "optimal"

$d^* = 44$ $d^* = 38$ $d^* = 32$ $d^* = 26$ $d^* = 20$ $d^* = 14$ $d^* = 8$

Fig.2   MBRG region maps vs. spectral distance threshold at 16 band resolution.

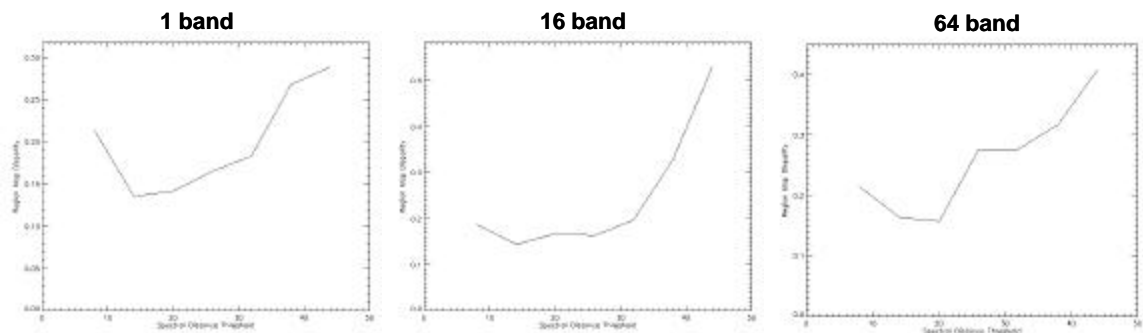**1 band**   **16 band**   **64 band**

Fig.3   Plots of disparity between MBRG region maps and the calibration edge
map vs. d^* for the AVIRIS image block in Fig.2 at various spectral
resolutions.

Fig.4 provides "optimal" MBRG segmentations at 6 different spectral resolutions for
each of 3 AVIRIS image blocks. AVIRIS1 is built-up (busy). AVIRIS2 is semi-rural. AVIRIS3
is rural. For each image, the region maps appear to be subjectively similar across all spectral
resolutions. Fig.5 shows calibration edge maps extracted automatically for each of these 3
images, along with reference edge maps generated manually by the Author. The calibration edge
maps, though generated with a very simple algorithm, are quite similar to the manually generated
reference edge maps (but the calibration edge maps are busier). For each image, disparity vs. the
calibration edge map is plotted at 6 spectral resolutions. Disparity vs. the reference edge map is
computed at the same 6 spectral resolutions. These plots clearly establish that there is not
generally a strong correlation between spectral resolution and segmentation quality. They also
show that disparity is a function of scene content (busier images tend to have smaller disparities).
The plots show that it is not necessary to supply manually generated reference edge maps.
Automatically generated calibration edge maps are good enough, which is important since a
major objective is to automate the calibration process. However, because the manually generated
edge maps tend to be more sparse, one can expect disparity to be larger than for the automatically
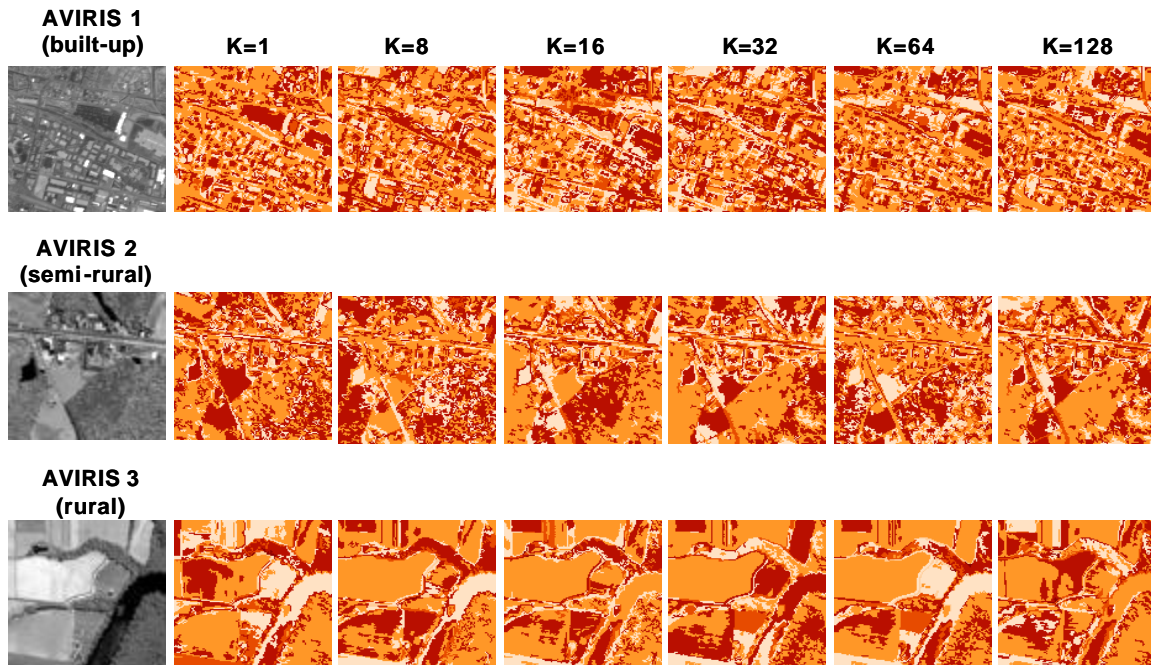generated calibration edge maps.

Fig.4 "Optimal" MBRG segmentations for 3 AVIRIS image blocks at 6 spectral resolutions.
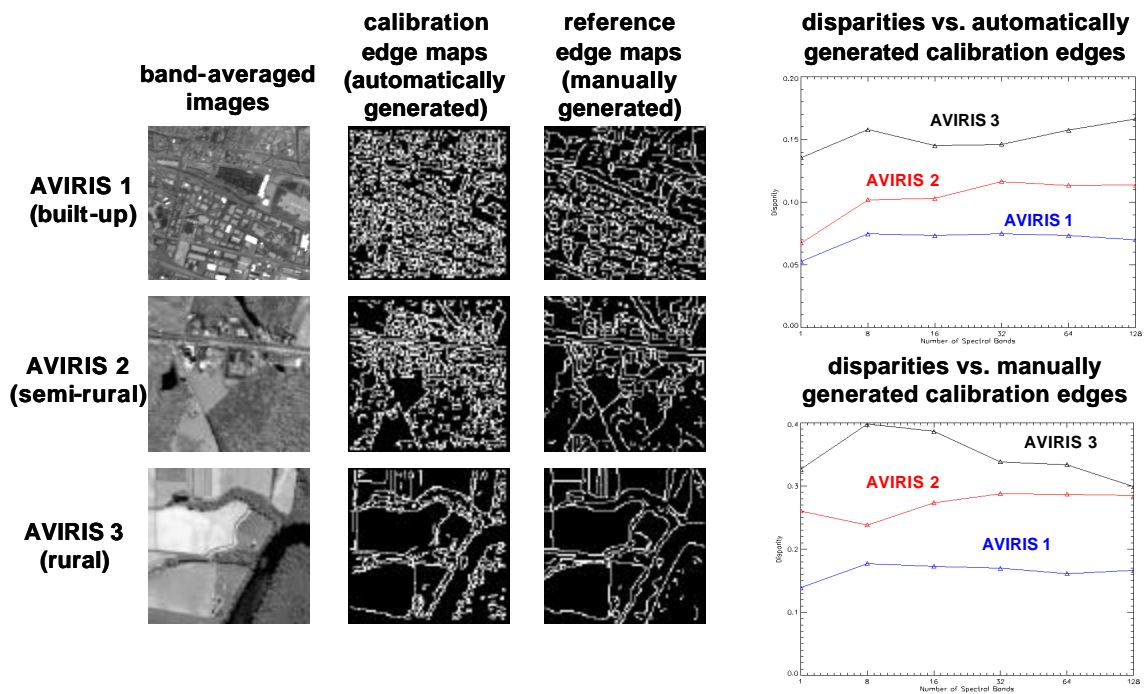

Fig.5 Automatically generated calibration edge maps and manually generated reference edge maps for 3 AVIRIS images. Region map disparities vs. each edge map at 6 spectral resolutions.

6. Summary and Future Directions

A novel self-calibration framework was introduced for automatically selecting algorithm parameter values producing image segmentations that most closely resemble a calibration edge map (derived separately and automatically using a simple edge detector). The framework is spatially adaptive in that parameter values can vary for different image blocks, and generic in that it can imbed any core segmentation algorithm. The Multi-Band Region Growing (MBRG) algorithm was developed as a generalization of the classical region growing approach to segmentation from single to multi-band images, and was then introduced to the self-calibration framework as the core image segmentation algorithm. The framework was applied to AVIRIS image blocks corresponding to scenes with varying degrees of busyness, and at different spectral resolutions. It was shown that both subjectively and quantitatively, image segmentation quality does not generally appear to be highly correlated with spectral resolution. Since the computational complexity of image segmentation increases with the number of spectral bands, it may thus often be best to apply segmentation to images that have been blurred to modest spectral resolution (say to tens of spectral bands).

The current self-calibration strategy selects the region map that most closely resembles an independently derived calibration edge map. Although this approach does usually produce reasonable and appropriate settings for image segmentation algorithm parameters, it does not take into account the fact that there is often no one setting that produces a region map that contains most or all regions and objects of interest. In practice, the majority of regions and objects of interest often do not lie in any one region map, but across several. This suggests an important topic for future research, namely development of a multi-scale technique for merging region maps at different levels of coarseness and granularity corresponding to different settings on the segmentation algorithm parameters.

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